Q5/S4 $\vec{F} = yz\log x\,\vec{i} + zx\log y\,\vec{j} + xy\log z\,\vec{K}$

$$\text{Curl}\,\vec{F} = \vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz\log x & zx\log y & xy\log z \end{vmatrix}$$

$$= \vec{i}\left[\frac{\partial}{\partial y}(xy\log z) - \frac{\partial}{\partial z}(zx(\log y))\right]$$
$$+\vec{j}\left[\frac{\partial}{\partial z}(yz\log x) - \frac{\partial}{\partial x}(xy\log z)\right]$$
$$+\vec{K}\left[\frac{\partial}{\partial x}(zx(\log y)) - \frac{\partial}{\partial y}(yz\log x)\right]$$

$$= [x\log z - x\log y]\,\vec{i} + [y\log x - y\log z]\,\vec{j} + [z\log y - z\log x]\,\vec{K}$$

$$= x(\log z - \log y)\,\vec{i} + y(\log x - \log z)\,\vec{j} + z(\log y - \log x)\,\vec{K}$$

$$\text{Curl}\,\vec{F} = x\log\frac{z}{y}\,\vec{i} + y\log\frac{x}{z}\,\vec{j} + z\log\frac{y}{x}\,\vec{K}$$

Ans

v.v.imp

(vii / 84) Prove $\vec{\nabla} \times (b\vec{f} + c\vec{g}) = b\vec{\nabla} \times \vec{f} + c\,\vec{\nabla} \times \vec{g}$

Proof: $\vec{f} = [f_1, f_2, f_3]$, $\vec{g} = [g_1, g_2, g_3]$

$$\vec{\nabla} \times (b\vec{f} + c\vec{g}) = \vec{\nabla} \times \big[b[f_1, f_2, f_3] + c[g_1, g_2, g_3] \big]$$

$$= \vec{\nabla} \times \big[[bf_1, bf_2, bf_3] + [cg_1, cg_2, cg_3] \big]$$

$$= \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right] \times \left[bf_1 + cg_1, bf_2 + cg_2, bf_3 + cg_3 \right]$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ bf_1 + cg_1 & bf_2 + cg_2 & bf_3 + cg_3 \end{vmatrix}$$

By R_1

$$= \vec{i}\left[\frac{\partial}{\partial y}(bf_3 + cg_3) - \frac{\partial}{\partial z}(bf_2 + cg_2) \right] + \vec{j}\left[\frac{\partial}{\partial z}(bf_1 + cg_1) - \frac{\partial}{\partial x}(bf_3 + cg_3) \right] + \vec{k}\left[\frac{\partial}{\partial x}(bf_2 + cg_2) - \frac{\partial}{\partial y}(bf_1 + cg_1) \right]$$

$$= \vec{i}\left[b\frac{\partial f_3}{\partial y} + c\frac{\partial g_3}{\partial y} - b\frac{\partial f_2}{\partial z} - c\frac{\partial g_2}{\partial z} \right]$$

$$+ \vec{j}\left[b\frac{\partial f_1}{\partial z} + c\frac{\partial g_1}{\partial z} - b\frac{\partial f_3}{\partial x} - c\frac{\partial g_3}{\partial x} \right]$$

$$+ \vec{k}\left[b\frac{\partial f_2}{\partial x} + c\frac{\partial g_2}{\partial x} - b\frac{\partial f_1}{\partial y} - c\frac{\partial g_1}{\partial y} \right]$$

$$= \vec{i}\left[b\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right)\right] + \vec{i}\left[c\left(\frac{\partial g_3}{\partial y} - \frac{\partial g_2}{\partial z}\right)\right]$$

$$+ \vec{j}\left[b\left(\frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x}\right)\right] + \vec{j}\left[c\left(\frac{\partial g_1}{\partial z} - \frac{\partial g_3}{\partial x}\right)\right]$$

$$+ \vec{k}\left[b\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) + \vec{k}\left[c\left(\frac{\partial g_1}{\partial z} - \frac{\partial g_1}{\partial x}\right)\right]\right]$$

$$= b\left[\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right)\vec{i} + \left(\frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x}\right)\vec{j} + \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)\vec{k}\right]$$

$$+ c\left[\left(\frac{\partial g_1}{\partial y} - \frac{\partial g_2}{\partial z}\right)\vec{i} + \left(\frac{\partial g_1}{\partial z} - \frac{\partial g_3}{\partial x}\right)\vec{j} + \left(\frac{\partial g_2}{\partial x} - \frac{\partial g_1}{\partial x}\right)\vec{k}\right)$$

$$\vec{\nabla} \times (b\vec{f} + c\vec{g}) = b\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix} + c\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ g_1 & g_2 & g_3 \end{vmatrix}$$

$$\vec{\nabla} \times (b\vec{f} + c\vec{g}) = b\,\vec{\nabla} \times \vec{f} + c\,\vec{\nabla} \times \vec{g}.$$ Proved

v.imp (viii) prove $\vec{\nabla} \times (\phi \vec{f}) = \phi \, \vec{\nabla} \times \vec{f} + (\vec{\nabla} \phi) \times \vec{f}$.

proof:- $\vec{f} = [f_1, f_2, f_3]$

$$\vec{\nabla} \times (\phi \vec{f}) = \vec{\nabla} \times [\phi . [f_1, f_2, f_3]].$$

$$= \vec{\nabla} \times [\phi f_1, \phi f_2, \phi f_3]$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \phi f_1 & \phi f_2 & \phi f_3 \end{vmatrix}$$

$$= \vec{i}\left[\frac{\partial}{\partial y}(\phi f_3) - \frac{\partial}{\partial z}(\phi f_2)\right] + \vec{j}\left[\frac{\partial}{\partial z}(\phi f_1) - \frac{\partial}{\partial x}(\phi f_3)\right]$$

$$+ \vec{K}\left[\frac{\partial}{\partial x}(\phi f_2) - \frac{\partial}{\partial y}(\phi f_1)\right]$$

$$\left.\begin{aligned}
&= \vec{i}\left[\phi\frac{\partial f_3}{\partial y} + \frac{\partial \phi}{\partial y} f_3 - \phi\frac{\partial f_2}{\partial z} - \frac{\partial \phi}{\partial z} f_2\right] \\
&\quad + \vec{j}\left[\phi\frac{\partial f_1}{\partial z} + \frac{\partial \phi}{\partial z} f_1 - \phi\frac{\partial f_3}{\partial x} - \frac{\partial \phi}{\partial x} f_3\right] \\
&\quad + \vec{K}\left[\phi\frac{\partial f_2}{\partial x} + \frac{\partial \phi}{\partial x} f_2 - \phi\frac{\partial f_1}{\partial y} - \frac{\partial \phi}{\partial y} f_1\right]
\end{aligned}\right\} (1)$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F} = \vec{i}\left[\frac{\partial}{\partial y} f_3 - \frac{\partial f_2}{\partial z}\right] + \vec{j}\left[\frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x}\right] + \vec{K}\left[\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right]$$

$$\left.\begin{aligned}
\phi \vec{\nabla} \times \vec{F} &= \vec{i}\left[\phi\frac{\partial f_3}{\partial y} - \phi\frac{\partial f_2}{\partial z}\right] + \vec{j}\left[\phi\frac{\partial f_1}{\partial z} - \phi\frac{\partial f_3}{\partial x}\right] \\
&\quad + \vec{K}\left[\phi\frac{\partial f_2}{\partial x} - \phi\frac{\partial f_1}{\partial y}\right]
\end{aligned}\right\} (2)$$

$$(\vec{\nabla}\phi) \times \vec{F} = \left[\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right] \times \left[f_1, f_2, f_3\right]$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$(\vec{\nabla}\phi)\times\vec{F} = \vec{i}\left[f_3\frac{\partial\phi}{\partial y} - f_2\frac{\partial\phi}{\partial z}\right] + \vec{j}\left[f_1\frac{\partial\phi}{\partial z} - f_3\frac{\partial\phi}{\partial x}\right] + \vec{K}\left[f_2\frac{\partial\phi}{\partial x} - f_1\frac{\partial\phi}{\partial y}\right] \tag{3}$$

(2) + (3)

$$\begin{aligned}\phi\vec{\nabla}\times\vec{F} + (\vec{\nabla}\phi)\times\vec{F} = {} & \vec{i}\left[\phi\frac{\partial f_3}{\partial y} + \frac{\partial\phi}{\partial y}f_3 - \phi\frac{\partial f_2}{\partial z} - \frac{\partial\phi}{\partial z}f_3\right] \\ & + \vec{j}\left[\phi\frac{\partial f_1}{\partial z} + \frac{\partial\phi}{\partial z}f_1 - \phi\frac{\partial f_3}{\partial x} - \frac{\partial\phi}{\partial x}f_3\right] \\ & + \vec{K}\left[\phi\frac{\partial f_2}{\partial x} + \frac{\partial\phi}{\partial x}f_2 - \phi\frac{\partial f_1}{\partial y} - \frac{\partial\phi}{\partial y}f_1\right]\end{aligned} \tag{4}$$

(1) = (4)

So

$$\vec{\nabla}\times(\phi\vec{F}) = \phi\,\vec{\nabla}\times\vec{F} + (\vec{\nabla}\phi)\times\vec{F}.$$

<u>proved</u>